Working Paper

Document de travail

Fiscal Policy and
Economic Analysis
Branch

Department of Finance

Direction de la
politique fiscale
et de l'analyse économique

Ministère des Finances

101-06190/

The Structure of Production in Ten Canadian Industries

by

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Working Paper No. 86-1

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This paper was presented at the 1986 Meetings of the Canadian Economics Association, University of Manitoba. The authors wish to thank Alexandra Cas from Statistics Canada and Someshwar Rao from the Economic Council of Canada for providing the data and for helpful discussions. In writing this paper, we also have benefited from comments by Mario Albert, Jeffrey Bernstein, Erwin Diewert, Melvyn Fuss, Lorey Hoffman, Paul-Henri Lapointe, Kevin Lynch, Jack Mintz, Benoît Robidoux, Andrew Sharpe and John Whalley. Expert technical assistance by Richard Hotte is also acknowledged. The authors retain sole responsibility for any remaining errors.

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RESUME

Le présent document présente une analyse économétrique de la structure de production de dix industries canadiennes. L'analyse statique utilise une fonction de coût et des techniques économétriques assez courantes. L'analyse comporte deux aspects inhabituels: premièrement, une distinction est faite dans le cas du stock de capital entre les équipements et les structures; secondement, les inventaires de matières intermédiaires sont inclus comme facteur de production. Ces caractéristiques permettent une analyse plus complète de l'impact des changements dans les coûts des facteurs de production.

Les principales conclusions de l'analyse empirique sont les suivantes:

1) Le capital est séparable des autres facteurs de production dans quatre industries sur dix seulement. Cela signifie qu'il est important de désagréger le capital en structures et équipements car, dans la majorité des industries examinées, les changements dans le prix relatif des structures par rapport aux équipements influencent la demande de tous les facteurs de production même si le prix du capital agrégé est constant.

2) L'inclusion des inventaires de matières intermédiaires comme facteur de production est importante car elle accroît la capacité d'explication des équations et entraîne des changements importants sur le plan des possibilités de substitution des autres facteurs de production.

ABSTRACT

This paper presents an econometric analysis of the structure of production in ten Canadian industries. The analysis is undertaken using a static cost function and fairly standard econometric techniques. There are two unusual features of the analysis: capital is disaggregated into equipment and structures; and inventories of materials are included as a factor of production. These features facilitate a more complete analysis of the impact of changes to the costs of factors of production.

The key empirical findings are:

(1) Capital is separable from other inputs in only four of the ten industries examined. This implies disaggregation of the capital stock is important because, in the majority of industries examined, changes in the relative price of equipment and structures influence the demand for all factors of production even if the price of aggregate capital is unchanged.

(2) Including inventories as a factor of production is important since it increases the explanatory power of the equations and causes significant changes in the substitution patterns among other factors of production.

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Introduction

This paper presents an econometric analysis of the structure of production in ten Canadian industries. There are two unusual features of the analysis: capital is disaggregated into equipment and structures; and inventories of materials are included as a factor of production. Capital is rarely disaggregated in studies of this sort. To our knowledge, there is only one Canadian multi-sector study in which capital is separated into equipment and structures, and this study is somewhat dated.[1] Disaggregating capital would be of minor importance if, for a given industry, all cross elasticities of substitution involving equipment or structures on one hand and the remaining inputs on the other hand were the same. In a model with equipment, structures and labour as factors of production, this would imply that the elasticity of substitution beween labour and equipment is the same as between labour and structures. This is clearly an empirical issue. Formal separability tests undertaken in this paper indicate that cross elasticities of substitution involving equipment or structures and other inputs are not the same in six of the industries examined. In other words, equipment and structures can be combined to form a consistent aggregate in only four industries.

There do not appear to be any studies that include inventories as a factor of production, although Diewert (1985) has argued that they should be so considered. Firms hold inventories of materials, goods in process and finished products in order to permit planned production to take place more efficiently. In addition inventories of finished goods perform a significant buffering role for unexpected changes in demand. Modelling this disequilibrium effect is beyond the scope of this paper since it would require a full-fledged dynamic model; so only inventories of materials and goods in process are included in the analysis.[2] The

1. See Woodland (1975). His reported results are for 1961.

2. For a recent analysis of the role of inventories in price and output adjustments, see Blinder (1982) and Blanchard (1983). For an application within a complete production framework, see Helliwell and Chung (1985).

substitution possibilities between inventories and other inputs are of interest in themselves; in addition the inclusion of inventories alters the estimated substitution elasticities for all inputs.

The remainder of this paper is organized as follows: section 1 provides a detailed description of the model used in this study. The data and the methodological considerations pertinent in their construction are described in section 2. Section 3 describes the econometric technique used to estimate the model and reviews some alternative approaches. Sections 4 and 5 provide a detailed discussion of the empirical results and separability tests for the ten sectors examined. A comparison with different models is made in section 6. Our findings and conclusions are summarized in the final section.

1. The Underlying Production Technology: The Translog Cost Function

In this section, we describe the structural model used to analyse the structure of production in ten Canadian industries. For this purpose, a production, cost or profit function framework could be used.[3] The cost and profit function approaches are preferable on econometric grounds since it is assumed that input prices rather than quantities are more properly considered exogenous.

The cost function framework is appropriate when input prices and output are exogenous and when firms minimize costs while in a profit function framework output is endogenously chosen to maximize profits. Both cost and profit functions have been used in studies of Canadian industry. However, empirical work by Artus (1983) supports the cost-minimization approach, which is used in this study. The cost function framework would be less reasonable if inventories of finished goods were included in the model. Output and finished goods inventories are simultaneously determined, so output would be more properly considered an endogenous variable in this case, which would imply use of a profit function framework.

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3. Note that the "duality" of cost and production functions is not necessarily respected in the case of the translog flexible form. For more details, see Burgess (1975), Applebaum (1978), Caves and Christensen (1980) and Geary and McDonnell (1980).

The next step is to choose a functional form of the cost function.[4] Diewert (1971) pioneered the concept of flexible functional forms. His seminal work was pursued by others such as Christensen, Jorgenson and Lau who introduced the translog function, which[5] does not impose a priori restrictions on elasticities of substitution. Considered inter alia as a direct generalization of the Cobb-Douglas and C.E.S. functions, the translog permits straightforward computation of the elasticities of substitution and price elasticities.

We can write the general form of the translog cost function

$$\begin{aligned} \ln C = {} & \alpha_o + \alpha_Q \ln Q + \tfrac{1}{2}\gamma_{QQ}(\ln Q)^2 + \gamma_{Qt} \ln Q \cdot t \\ & + \sum_i \alpha_i \ln P_i + \tfrac{1}{2}\sum\sum_{ij} \gamma_{ij} \ln P_i \ln P_j \\ & + \sum_i \gamma_{Qi} \ln Q \ln P_i + \alpha_t \cdot t \qquad (1) \\ & + \tfrac{1}{2}\gamma_{tt} \cdot t^2 + \sum_i \gamma_{ti} \ln P_i \cdot t \end{aligned}$$

where Q : real gross output;

C : total cost;

P_i : price of input i and $i = K^S, K^E, L, E, M, I$;

t : proxy for technical change;

K^S : structures;

K^E : equipment;

L : labour;

E : energy;

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4. Jorgenson (1985) reviews some of the literature on applications of flexible forms to production and cost functions.

5. Christensen, L.R., Jorgenson, D.W. and Lau, L. (1971) "Conjugate Duality and the Transcendental Logarithmic Production Function" Econometrica, Vol. 39, No. 4, pp. 225-6.

M : materials;

I : inventories of materials.

The following restrictions should be applied on (1) in order to get a "well-behaved" cost function[6]:

$\Sigma_i \alpha_i = 1$ (Identifiability of distribution parameters)

$\Sigma_i \gamma_{Qi} = 0$ (Engel aggregation)

$\Sigma_i \gamma_{ti} = 0$

$\Sigma_j \gamma_{ij} = 0$ (Cournot aggregation)

$\Sigma_i \gamma_{ji} = 0$ (Cournot aggregation)

$\gamma_{ij} = \gamma_{ji}$ (Slutsky symmetry)

If we assume that markets are competitive, a necessary set of conditions for efficient production are the following cost share equations (S_i) implied by Shephard's lemma (Varian (1978), p. 32) which, in the translog case, have the useful characteristics of being linear in the parameters:

$$S_i = \frac{\partial \ln C}{\partial \ln P_i} = \frac{P_i X_i}{C} = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \gamma_{ti} \cdot t + \gamma_{Qi} \ln Q \qquad (2)$$

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6. These restrictions guarantee the adding up conditions ($\sum_i S_i = 1$ since $\sum_i P_i X_i = C$) and, with concavity and monotonicity conditions, that the function will be well behaved (or satisfy the requirements of neo-classical production theory). Since the translog cost function is a second-order approximation to any continuous twice differentiable cost function, the matrix of γ_{ij}'s in (1) is equivalent to the Hessian of the Taylor series expansion of the "true" cost function. Thus, by Young's theorem ($\gamma_{ij} = \gamma_{ji}$), this matrix must be symmetric. Moreover, if the log-quadratic terms are ignored ($\gamma_{ij}=0$ $i=K^S, K^E, L, E, M, I$), the translog becomes simply a Cobb-Douglas function. A similar transformation can be done to obtain the C.E.S. functional form. Denny and Fuss (1977) provide extensive discussions of these topics. See Hanoch (1975) for the classification of the restrictions.

where X_i is the quantity of factor i.

The system of equations (2) represents the core of the econometric exercise. Since any deviations of the shares from the logarithmic marginal products are assumed to occur as errors in cost minimizing behaviour, we specify (2) appended by additive normal disturbances. The theory of cost and production requires that total costs increase monotonically with all input prices and that the input requirement set be locally convex. Violation of these regularity conditions would indicate a potential specification problem in our translog cost framework. This possibility is assessed in the section on empirical results.

Following Berndt and Wood (1975), the estimation of the system of equations (2) permits the computation of the Hicks-Allen partial elasticities of substitution (σ_{ii}, σ_{ij}) as well as the own- and cross-price elasticities (ε_{ii}, ε_{ij}):

$$\sigma_{ij} = (\gamma_{ij}/S_iS_j) + 1 \qquad \text{for } i \neq j$$

$$\sigma_{ii} = (\gamma_{ii} + S_i^2 - S_i)/S_i^2 \qquad \text{for all } i$$

$$\varepsilon_{ij} = S_j\sigma_{ij} \qquad \text{for } i \neq j$$

$$\varepsilon_{ii} = S_i\sigma_{ii} \qquad \text{for all } i$$

Homotheticity of the implicit production structure described by (1) is guaranteed if all $\gamma_{Qi}=0$.[7] In this case, the cost shares (2) are a function of the input prices and do not depend on the output level. Non-homotheticity would occur if one (or more) $\gamma_{Qi}\neq 0$. In addition, given $\alpha_Q=k$ and $\gamma_{QQ}=\gamma_{Qt}=0$, the cost function would be homogenous of degree k and if k=1, the function would exhibit constant returns to scale.

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7. The concept of homotheticity was introduced by Shephard (1953), who demonstrated the equivalence of homotheticity conditions between cost and production functions.

In this model, if $\gamma_{ti}=0$ for all i, technical change is said to be neutral, which means that it does not affect factor shares. Technical progress can be described as "factor using" or "factor saving" depending of the sign of γ_{ti}.[8] For instance, technical change would be structures using (biased toward structures) if $\gamma_{tK}S>0$, which implies an increase in the share of structures in total costs over time with all input prices constant. In this case, an increase in the user cost of structures encourages substitution of other inputs for structures, but it also reduces the rate of technical progress.[9]

2. Data

With the exception of inventories, the data used in this study are from a data base developed at the Economic Council of Canada by Someshwar Rao. The data base was developed making use of several sources; these are indicated below. The inventory data were developed by Alexandra Cas at Statistics Canada.

The real output measure used in this study is gross output as estimated by the Industry Product Division, Statistics Canada. The price and quantity of materials are also from the Industry Product Division. Materials are defined to include raw materials, components, other supplies used up in the production process as well as purchased service inputs such as transportation, advertising and insurance. The price of materials is the output price of the supplying industries adjusted to include indirect taxes.

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8. A review of the literature on biased technical change can be found in Binswanger (1978).

9. This can be seen more clearly by taking the partial derivative of equation (1) with respect to time:

$$\frac{\partial \ln C}{\partial t} = \alpha_t + \gamma_{Qt} \cdot \ln Q + \sum_i \gamma_{ti} \ln P_i + \gamma_{tt} \cdot t.$$

This derivative is expected to be negative so if $\gamma_{tk}s > 0$, technical progress decelerates. Note that the sum of the γ_{ti} is zero.

Data on current dollar energy consumption in manufacturing is taken from the General Review of the Manufacturing Industries (31-203). To obtain constant dollar consumption of energy, a time series on energy prices by industry is needed. Since this is not readily available, Rao and Preston (1983) constructed energy price indices making use of data on gross output prices of these industries. These price indices include indirect taxes. The "energy" industries are coal, crude petroleum and natural gas, petroleum and coal products and utilities. An energy price index was constructed for manufacturing industries using deliveries of the energy industries as weights. It is worth noting that total energy use in manufacturing calculated by this method is approximately equal to the estimates of purchased fuel and electricity found in the Census of Manufactures.

The quantity of labour is defined in terms of manhours worked as calculated by the Productivity Section, Statistics Canada. The price of labour is calculated as the sectoral wage bill, which includes wages, salaries and supplementary labour income, divided by hours worked.

The capital stock by industry was calculated using benchmark capital stocks, assumed service lives and investment flows supplied by the Construction Division, Statistics Canada. Estimates of the price of depreciable capital services are from Lodh (1984). The user cost formula used by Lodh is essentially the Hall and Jorgenson (1967) approach modified to take into account sector-specific depreciation rates, tax parameters, and debt-equity ratios.

Inventories of materials are from Cas (1984). They consist of commodities purchased (raw materials, components and other supplies), but not yet used in the production process as well as goods in process. This latter item is also included in output. The outstanding stock is valued at current period prices. The price deflator for inventories was calculated by Cas (1984) using the same methodology as for materials.

We constructed a measure of the user cost of inventories based on the Boadway, Bruce and Mintz (1982) methodology. The rationale underlying their approach is the same as for depreciable capital:

firms will increase inventory (capital stock) provided that the net-of-tax marginal benefits exceed marginal cost. Assuming FIFO inventory valuation, the user cost for inventories (ψ) is determined by the following formula:

$$\psi = \frac{r_f + u(p^e - v)}{(1-u)}$$

where

r_f: the after-tax cost of financing for the firm;
u: the combined federal-provincial corporate tax rate;
p^e: the expected rate of inflation;
v: rate of inventory allowance.

The after-tax cost of financing (r_f) is a weighted average of the after-tax return on debt and the before-tax return on equity. Following Boadway, Bruce and Mintz (1982), it is assumed that the CPI is the appropriate reference price. The expected inflation rate is from the CANDIDE model, version 3.0. It is a stochastic equation with past rates of inflation and past rates of change in the money supply as explanatory variables. (This expected rate is also used to calculate the user cost of depreciable capital.) The rental price is obtained as the product of the user cost and the implicit deflator for inventories.

Defining total cost is problematic since it is unobservable. One solution is to equate the observable value of gross output with total cost. This approach is predicated on the assumption of constant returns to scale and competitive behaviour in product and factor markets. However, these assumptions may not be a useful representation of the situation in the Canadian economy. Fuss (1977) used a markup model in which it is assumed that total cost is a constant or smoothly changing percentage of gross output in current dollars. This approach is also adopted by Rao and Preston (1983). Two remarks should be made regarding this pricing model: (i) the markup cannot be separated from Hicks-neutral technical progress and (ii) the inclusion of an unidentified markup in costs means that one of the factor payments must be determined residually in order for the factor shares to sum to unity.

If the markup is indeed constant or smoothly changing, calculating the cost of capital as a residual would not cause any econometric problems. However, the markup, defined as one minus the ratio of factor payments to the value of output, is variable.

In this paper total cost is defined as the sum of payments to factors of production using an expected rate of return and the observed stock of capital to calculate payments to capital. Actual payments to capital may differ from this measure because of oligolopolistic markups and cyclical variations in profitability.

3. Estimation of the Cost Function

The system of stochastic equations (2) was estimated using annual data for ten Canadian industries for the period 1961 to 1980. Since the cost shares of the five equations in (2) (six for manufacturing) always sum to unity[10], the sum of the disturbances across the five equations is zero at each observation. Therefore, the disturbance variance-covariance matrix is singular and nondiagonal. The adding-up conditions with the symmetry restrictions ($\gamma_{ij} = \gamma_{ji}$) guarantee that any four of the equations (2) will exactly identify all the parameters of the system. Thus, we drop one equation and one random term. The disturbance column vector for the remaining error terms is specified to be independently and identically normally distributed with mean zero and nonsingular variance-covariance matrix.

10. Since the shares are constrained to the unit simplex (bounded between 0 and 1), the multivariate Gauss-Laplace distribution assumption implies that there is a positive probability that a share (or more) might not satisfy this constraint. As Woodland (1979) pointed out, it is then technically correct to state that the normality assumption is invalid as a stochastic specification for the purpose of the estimation of share equations. Performing counterfactual experiments using the Dirichlet distribution (which constraints the shares to lie on the unit simplex), Woodland's Monte Carlo analysis indicates that estimation results from the Gauss-Laplace and Dirichlet models are very close. Consequently, there does not appear to be a particular need to worry about the empirical consequences of the underlying normality hypothesis.

The system of equations (2) could be estimated with constraints by performing a four-equation multivariate OLS regression that minimizes the trace of the variance-covariance matrix. However, the Zellner method leads to an improvement in efficiency by taking into account the non-zero cross-equation error correlations generated by the constraints across share equations. The problem with this method is that the parameters can change depending on which equation is dropped. As shown by Barten (1969), maximum likelihood estimation leads to parameter estimates which are invariant to the equation dropped for the estimation exercise. Kmenta and Gilbert (1968) and Ruble (1968) have shown that full information maximum likelihood (FIML) and iteration of the Zellner technique are computationally equivalent in terms of efficiency. The iterative Zellner technique is used here.[11]

Since we use aggregate data, it has been argued that factor prices (and quantities) should be treated as endogenous variables.[12] To handle this problem, it is necessary to specify factor price equations and estimate the system of equations simultaneously.[13] While a 2SLS procedure would correct the bias, the cross-equation restrictions cannot be applied using this method. Efficiency can be gained by using 3SLS, i.e. performing the Zellner technique to a set of equations estimated by 2SLS. As shown by Berndt and Savin (1975), the 3SLS can be iterated (I3SLS) in order to avoid the dependence on the retained estimated equations. Dhrymes (1973) found that the iterative 3SLS technique is asymptotically equivalent to the FIML estimator of the simultaneous

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11. See Aigner (1973) for an exhaustive description of estimation techniques for these models.

12. Berndt and Christensen (1973), p. 89, Berndt and Wood (1975), p. 261 and Burgess (1975), p. 109.

13. For example, the exogenous variables used by Berndt and Christensen (1973) for their price equation system are the U.S. population, U.S. population of working age, the effective rate of sales and excise taxation, the effective rate of property taxation, government spending variables, real exports of non-durable goods and services and the U.S. tangible capital at the end of the previous year. Burgess (1975) used a subset of these variables in order to perform I3SLS.

model. However, in light of the additional data requirements the I3SLS technique was not applied. Moreover, Berndt and Christensen (1973) found that I3SLS did not significantly modify the results. As Denny and May (1977), we expect that same situation would prevail for Canadian data.

4. Empirical Results

The results of the econometric exercise can be summarized briefly as follows:

1. For most industries about 88% of the variation in the share equations (2) is explained by the variation in relative input prices, output levels and technical change.

2. For all industries the cost function is found to be non-homothetic, which means that input proportions vary with the level of output, everything else equal.

3. Technical change is found to be non-neutral in all industries, and is capital using in almost all industries.

4. For every sector, the monotonicity conditions described in the previous section are verified although there are local concavity problems in some sectors. In forestry, construction, manufacturing durables and non-durables and utilities, violation of concavity conditions is statistically significant for some observations.[14]

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14. Concavity is verified by calculating the characteristic roots at each observation from the Hessian matrix which is formed by the second-order partial derivatives of the cost function with respect to the exogenous variables. Concavity requires that the characteristic roots be non-positive. It can be formally shown that if the characteristic root is positive then the associated own elasticity of substitution (and the own-price elasticity) will be positive. However, negative own-price elasticities do not guarantee the satisfaction of the concavity requirements. In other words, negative own-price elasticities are only necessary conditions for the concavity of the cost function. See Morey (1986) for a thorough analysis of these topics.

5. The demand for factor inputs is generally price-inelastic: almost all own-price elasticities are smaller than unity in absolute value.

6. Substitution possibilities are relatively good in every sector; however structures and equipment are found to be complements in most sectors. Labour and the two types of depreciable capital are substitutes in a majority of industries in 1971 and 1979. Energy and structures are substitutes in durable manufacturing but complements in non-durable manufacturing. Energy and equipment are found to be substitutes in both manufacturing categories.

7. We conclude that structures and equipment are not separable from other inputs in construction, manufacturing non-durables, trade, transportation and communication, finance, insurance and real estate (henceforth FIRE) and utilities. In the remaining sectors equipment and structures form a consistent aggregate. Furthermore, in agriculture and mining, we cannot reject the hypothesis that all elasticities of substitution are unity (the Cobb-Douglas hypothesis cannot be rejected.) Finally, the existence of a value-added function is rejected in all sectors except mining.

8. In light of the changes observed in the substitution patterns, it is worthwhile to include inventories of materials as a separate input in the cost function. In addition the restriction that all coefficients on inventories of materials are zero is strongly rejected in all relevant sectors.

a) Econometric Results.

The econometric results as estimated by the iterative Zellner technique are presented in Table 1. In general, the results are quite satisfactory and the fits of the estimated shares are good.[15]

In Table 2, we report the estimated elasticities of substitution for the years 1971 and 1979. A number of own-price elasticities are positive, which causes the concavity problem noted in the fourth remark above. If this concavity condition is not satisfied, then the derived demand curve for the input will not be downward-sloping. As shown by Barnett and Lee (1984), the translog is particularly sensitive to this problem. Experiments performed by Wales (1977) designed to test the ability of the translog to approximate C.E.S. functions using data generated by an exact (non-stochastic) utility-maximizing process are pertinent in this context. He shows that even if the translog fails to meet the required regularity conditions, this does not imply the absence of an underlying optimizing process. Indeed his findings show that the fit of the translog share equation was usually very good (as measured by $\bar{R}^2$ or the divergence between the "true" and estimated price elasticity) even when the concavity conditions are mildly violated, implying that the parameters of the estimated cost function are close to their supposed true values. As Wales puts it: "A positive implication of this result, however, is that a violation of (concavity) conditions in practice does not necessarily preclude the obtaining of good price and income elasticity estimates" (p. 191).

Concavity conditions are often violated in empirical work using the translog cost function. For example, Woodland (1975) experienced the problem in the fishing sector for equipment while Caves,

15. The adjusted coefficient of determination ($\bar{R}^2$) for each equation is defined as $\bar{R}^2 = 1 - [\ \Sigma e^2/(n-k)/\Sigma y^2/(n-1)\ ]$ where e is the residual, n the number of observations, k the number of parameters and y the dependent variable in deviations. Except for ordinary least squares, $\bar{R}^2$ generally is not equivalent to the proportion of variance explained, since the orthogonality assumption of the residuals is violated. Nevertheless it still provides a fair measure of the goodness-of-fit.

TABLE 1

Econometric Results for the Static Translog Model, 1961-80

Estimated Coefficients		Agriculture	Forestry*	Construction	Manufacturing Durables	Manufacturing Non-durables	Trade	Transp. & Comm.	Fire	Utilities	Mining*
	α_I	0.0918	-	0.0111	0.0061	0.0150	0.0449	0.0096	0.0032	0.0066	-
		(5.68)		(10.73)	(1.65)	(3.50)	(1.70)	(5.86)	(1.41)	(1.55)	
	α_K	0.1379	0.0380	-0.0027	0.0198	0.0269	0.0426	0.1442	-0.0396	0.1272	0.0352
		(14.96)	(6.02)	(-4.06)	(7.86)	(7.11)	(15.96)	(9.86)	(-2.22)	(3.48)	(3.88)
	α_E	0.1992	0.0446	0.0117	0.0371	0.0508	0.0231	0.0771	-0.0056	0.1586	0.0446
		(6.27)	(9.12)	(2.83)	(7.13)	(8.31)	(9.35)	(6.68)	(-1.45)	(4.56)	(12.96)
	α_L	0.1288	0.3717	0.4185	0.7667	0.2208	0.4765	0.3559	0.0319	0.0676	0.3147
		(6.19)	(10.85)	(26.27)	(25.80)	(22.72)	(29.41)	(15.27)	(0.55)	(1.14)	(16.93)
	α_E	-	-	-	0.0094	0.0360					
					(5.31)	(17.95)					
	γ_{KS_KE}	-0.0144	-0.0339	-0.0042	-0.0023	-0.0058	-0.0044	-0.0117	-0.0011	0.0130	0.0156
		(-1.45)	(-6.04)	(-4.53)	(-1.35)	(-2.79)	(-1.97)	(-2.26)	(-1.00)	(0.88)	(3.21)
	γ_{KS_L}	-0.0023	-0.0319	-0.0057	-0.0068	0.0116	-0.0144	-0.0530	-0.0168	-0.0383	0.0055
		(-0.36)	(-4.27)	(-5.91)	(-2.95)	(4.79)	(-7.61)	(-8.53)	(-4.03)	(-4.02)	(0.81)
	γ_{KF_L}	-0.0408	0.0061	-0.0174	0.0011	0.0002	-0.0129	-0.0317	-0.0072	-0.0381	-0.0005
		(-2.57)	(1.06)	(-3.52)	(0.25)	(0.06)	(-6.84)	(-5.11)	(-2.67)	(-3.38)	(-0.17)
	γ_{KS_KS}	0.0853	0.0699	0.0053	0.0275	0.0316	0.0381	0.0953	0.0281	0.1802	0.0746
		(10.75)	(9.78)	(5.25)	(18.65)	(9.97)	(17.53)	(18.13)	(21.03)	(13.74)	(4.77)
	γ_{KF_KE}	0.1151	0.0374	0.0195	0.0435	0.0533	0.0309	0.0731	0.0106	0.0320	0.0423
		(3.58)	(7.00)	(5.08)	(13.13)	(18.77)	(12.06)	(11.08)	(2.88)	(1.61)	(13.19)
	γ_{LL}	0.0480	0.1796	0.1135	0.1244	0.0938	0.1803	0.1656	0.2142	0.1456	0.0090
		(3.35)	(7.83)	(5.82)	(11.16)	(10.00)	(13.82)	(13.43)	(6.16)	(8.52)	(0.65)
	γ_{KS_Q}	-0.0116	-0.0527	-0.0080	-0.0168	-0.0290	-0.0303	-0.0158	-0.0950	-0.3130	-0.1381
		(-1.03)	(-8.17)	(-5.81)	(-7.57)	(-5.37)	(-9.05)	(-1.09)	(-5.04)	(-9.98)	(-11.00)
	γ_{KF_Q}	0.0297	-0.0173	-0.0319	-0.0237	-0.0167	-0.0210	-0.0422	-0.0159	0.0201	-0.0521
		(0.44)	(-3.25)	(-3.55)	(-5.09)	(-1.85)	(-6.76)	(-3.69)	(-3.56)	(0.65)	(-11.16)
	γ_{LQ}	-0.0079	-0.2289	-0.0475	-0.0688	-0.0628	-0.0954	-0.1381	-0.4490	-0.2112	0.0248
		(-0.26)	(-6.98)	(1.38)	(-7.68)	(-3.96)	(-4.81)	(-5.98)	(-7.25)	(-4.12)	(0.96)
	γ_{KS_t}	-0.0147	0.0207	0.0059	0.0082	0.0143	0.0098	-0.0027	0.0982	0.3199	0.1730
		(-1.59)	(3.72)	(9.18)	(3.20)	(3.71)	(3.63)	(-0.19)	(5.67)	(8.85)	(21.39)
	γ_{KF_t}	0.0560	-0.0014	0.0200	0.0156	0.0125	0.0206	0.0490	0.0200	0.0000	0.0307
		(1.84)	(-0.31)	(5.00)	(2.96)	(2.05)	(8.28)	(4.35)	(5.24)	(0.00)	(9.90)
	γ_{Lt}	-0.0385	-0.0418	-0.0449	-0.0209	-0.0109	0.0279	0.0411	0.3226	0.1395	-0.0893
		(-1.87)	(-1.49)	(-2.94)	(-2.06)	(-1.12)	(1.70)	(1.80)	(5.61)	(2.39)	(-5.35)
	γ_{KS_I}	0.0000	-	-0.0014	0.0008	-0.0003	-0.0017	0.0000	-0.0002	-0.0052	-
		(-0.00)		(-1.16)	(0.53)	(-0.20)	(-0.91)	(0.00)	(-0.88)	(-2.42)	
	γ_{KF_I}	0.0303	-	0.0068	-0.0024	0.0034	-0.0004	-0.0011	0.0017	-0.0036	-
		(2.21)		(5.85)	(-0.97)	(1.74)	(-0.20)	(-1.24)	(2.48)	(-1.38)	
	γ_{LI}	0.0190	-	0.0031	-0.0236	-0.0076	0.0192	-0.0013	0.0038	0.0008	-
		(1.92)		(2.01)	(-6.37)	(-2.50)	(1.61)	(-1.44)	(2.22)	(0.51)	
	γ_{II}	0.0006	-	0.0048	0.0441	0.0155	-0.0074	0.0009	0.0012	0.0090	-
		(0.05)		(7.23)	(8.41)	(5.99)	(-0.47)	(2.28)	(0.27)	(10.38)	
	γ_{IQ}	-0.0486	-	0.0019	-0.0089	0.0019	-0.1014	0.0058	-0.00000	-0.0013	-
		(-2.01)		(0.86)	(-2.46)	(0.28)	(-3.29)	(3.53)	(-0.00)	(-0.34)	
	γ_{IT}	-0.0384	-	-0.0053	0.0085	-0.0030	0.0155	-0.0070	-0.0024	-0.0016	-
		(-2.19)		(-5.18)	(2.26)	(-0.65)	(0.58)	(-4.35)	(-1.08)	(-0.39)	
	γ_{IE}	-	-	-	0.0009	0.0027					
					(0.59)	(3.48)					
	γ_{KS_E}	-	-	-	0.0013	-0.0024					
					(1.86)	(-1.97)					
	γ_{KF_E}	-	-	-	0.0004	0.0029					
					(0.42)	(3.58)					
	γ_{LE}	-	-	-	-0.0015	-0.0017					
					(-0.87)	(-.193)					
	γ_{EE}	-	-	-	0.0113	0.0284					
					(11.64)	(29.85)					
	γ_{EQ}	-	-	-	-0.0090	0.0258					
					(-5.29)	(9.86)					
	γ_{ET}	-	-	-	0.0061	-0.0181					
					(3.35)	(-8.90)					
$\bar{R}^2$	S_KS	0.963	0.966	0.885	0.970	0.964	0.997	0.991	0.976	0.995	0.983
	S_KE	0.575	0.960	0.904	0.918	0.962	0.990	0.981	0.954	0.588	0.942
	S_L	0.569	0.965	0.853	0.926	0.940	0.967	0.906	0.715	0.866	0.900
	S_I	0.911	-	0.967	0.888	0.802	0.935	0.901	0.852	0.575	-
	S_E				0.936	0.995					
SSR	S_KS	0.00008	0.00029	0.000001	0.000009	0.000005	0.000006	0.000087	0.00007	0.000444	0.0007
	S_KE	0.00325	0.00018	0.000044	0.000042	0.000029	0.000005	0.000521	0.000004	0.000397	0.0000
	S_L	0.00068	0.01067	0.000660	0.000173	0.000083	0.000217	0.000222	0.0009	0.001133	0.0031
	S_I	0.00039	-	0.000003	0.000018	0.000016	0.0006	0.000001	0.000001	0.000006	-
	S_E	-	-	-	0.000004	0.000006					

* 1957-1982

TABLE 2

Own-Price Elasticities (ε), Hicks-Allen Elasticities of Substitution (σ) and Factor Shares (S)[1], 1971

Estimated Coefficients	Agriculture	Forestry	Construction	Manufacturing Durables	Manufacturing Non-durables	Trade	Trans. & Comm.	Fire	Utilities	Mining
ε_{LL}	-0.377	-0.103	-0.324	-0.245	-0.342	-0.141	-0.185	-0.037	-0.035	-0.726
ε_{MM}	-0.036	-0.122	-0.228	-0.065	0.005*	-0.045	-0.256	-0.053	0.289*	0.172*
$\varepsilon_{K^S K^S}$	-0.196	0.220*	0.617*	-0.011	-0.197	-0.233	-0.196	-0.466	-0.150	-0.434
$\varepsilon_{K^E K^E}$	-0.278	-0.004	-0.375	-0.139	-0.091	-0.249	-0.297	-0.239	-0.640	-0.361
ε_{II}	-0.928	-	-0.193	2.778	0.425*	-1.082	-0.702	-0.864	0.634	-
ε_{EE}	-	-	-	-0.260	0.593	-	-	-	-	-
σ_{LM}	0.454	0.149	0.567	0.405	0.299	0.017	0.392	0.036	-0.796	0.876
σ_{LK^S}	0.795	-0.753	-3.772	0.020	2.338	0.455	0.060	0.188	0.538	1.111
σ_{LK^E}	-0.908	1.518	-0.447	1.083	1.017	0.406	0.360	-0.460	-0.257	0.974
σ_{MK^S}	-0.125	0.887	4.106	-0.105	-0.272	0.074	0.369	0.708	-0.665	0.048
σ_{MK^E}	0.216	0.591	0.756	-0.156	-0.304	0.146	0.330	0.517	0.897	-0.598
$\sigma_{K^S K^E}$	0.517	-13.420	-37.955	-0.476	-1.206	-0.893	0.360	-0.346	1.182	1.991
σ_{IK^S}	1.000	-	-72.927	3.246	0.289	-0.397	1.007	-3.592	-1.137	-
σ_{IK^E}	3.046	-	35.808	-2.859	5.862	0.829	2.087	136.864	-3.002	-
σ_{IL}	4.384	-	2.440	-7.278	-2.334	1.719	-0.149	13.285	0.199	-
σ_{IM}	-0.652	-	-2.751	-1.612	-0.887	0.495	2.625	-9.536	1.554	-
σ_{EK^S}	-	-	-	3.934	-2.144	-	-	-	-	-
σ_{EK^E}	-	-	-	1.481	3.500	-	-	-	-	-
σ_{EL}	-	-	-	0.616	0.540	-	-	-	-	-
σ_{EM}	-	-	-	-0.241	-1.521	-	-	-	-	-
σ_{EI}	-	-	-	6.179	-0.887	-	-	-	-	-
S_K^S	0.126	0.060	0.003	0.029	0.041	0.053	0.144	0.059	0.442	0.210
S_K^E	0.238	0.039	0.033	0.054	0.063	0.044	0.127	0.014	0.162	0.075
S_L	0.090	0.302	0.363	0.243	0.209	0.494	0.390	0.349	0.187	0.236
S_I	0.062	-	0.006	0.012	0.011	0.054	0.003	0.001	0.005	-
S_M	0.484	0.599	0.594	0.647	0.658	0.355	0.335	0.577	0.203	0.479
S_E	-	-	-	0.015	0.018	-	-	-	-	-

(1) K^S: Structures; K^E: Equipment; L: Labour; M: Materials; I: Inventories of materials; E: Energy

* Not significantly positive at the 0.01 level of significance.

TABLE 2 (Cont'd)

Own-Price Elasticities (ε), Hicks-Allen Elasticities of Substitution (σ) and Factor Shares (S)[1], 1979

Estimated Coefficients	Agriculture	Forestry	Construction	Manufacturing Durables	Manufacturing Non-durables	Trade	Trans. & Comm.	Fire	Utilities	Mining
ε_{LL}	-0.456	-0.086	-0.326	-0.210	-0.292	-0.147	-0.186	-0.064	0.012	-0.779
ε_{MM}	-0.032	-0.102	-0.216	-0.038	0.032*	-0.076	-0.260	-0.064	-0.033	-0.165
$\varepsilon_{K^S K^S}$	0.031*	0.682	0.018*	0.537	0.065*	0.188*	0.111*	-0.352	-0.099	-0.453
$\varepsilon_{K^E K^E}$	-0.288	0.014*	-0.384	0.086*	0.181*	-0.088	-0.219	0.000	-0.637	-0.326
ε_{II}	-0.952	-	4.740	2.746	0.505*	-1.265	-0.135	-0.527	0.796	-
ε_{EE}	-	-	-	-0.400	0.131	-	-	-	-	-
σ_{LM}	0.592	0.150	0.562	0.377	0.232	0.160	0.497	0.104	-0.063	0.831
σ_{LK^S}	0.768	-1.021	-2.121	-0.738	3.149	0.066	-0.352	0.100	0.266	1.125
σ_{LK^E}	-0.484	1.558	-0.485	1.121	1.027	0.220	0.303	-0.695	-0.452	0.961
σ_{MK^S}	-0.432	0.085	2.894	-0.634	-0.603	-0.211	0.138	0.608	-0.348	0.273
σ_{MK^E}	0.313	0.606	0.767	-0.398	-0.617	0.145	0.306	0.321	0.939	-0.638
$\sigma_{K^S K^E}$	0.360	-19.797	-23.073	-2.021	-3.019	-2.803	-0.152	-1.256	1.288	1.846
σ_{IK^S}	1.000	-	-331.344	4.600	-0.064	-1.084	1.032	-19.252	-2.461	-
σ_{IK^E}	6.395	-	246.435	-3.952	8.163	0.548	-9.548	678.553	-3.715	-
σ_{IL}	8.607	-	11.749	-8.204	-3.332	2.622	-2.136	38.516	0.0510	-
σ_{IM}	-3.186	-	-25.104	-1.420	-0.923	0.130	5.214	-37.949	1.335	-
σ_{EK^S}	-	-	-	4.721	-2.001	-	-	-	-	-
σ_{EK^E}	-	-	-	1.531	3.397	-	-	-	-	-
σ_{EL}	-	-	-	0.655	0.619	-	-	-	-	-
σ_{EM}	-	-	-	0.069	-0.639	-	-	-	-	-
σ_{EI}	-	-	-	5.118	11.567	-	-	-	-	-
S_{K^S}	0.091	0.043	0.005	0.018	0.030	0.033	0.094	0.047	0.300	0.263
S_{K^E}	0.249	0.038	0.033	0.042	0.047	0.035	0.109	0.011	0.151	0.070
S_L	0.110	0.286	0.349	0.217	0.177	0.467	0.418	0.398	0.174	0.166
S_I	0.023	-	0.001	0.012	0.010	0.029	0.001	0.0002	0.005	-
S_M	0.527	0.633	0.612	0.692	0.710	0.439	0.378	0.544	0.370	0.500
S_E	-	-	-	0.019	0.026	-	-	-	-	-

(1) K^S: Structures; K^E: Equipment; L: Labour; M: Materials; I: Inventories of materials; E: Energy

* Not significantly positive at the 0.01 level of significance.

Christensen and Swanson (1981) found a positive (but statistically not significant) own-price elasticity for capital in U.S. railroads. Own-price elasticities estimated by Rao and Preston (1983) also indicate failure of concavity conditions for energy in manufacturing non-durables, non-metallic minerals, paper and allied products for 1971 and for materials in FIRE (1971 and 1979) and utilities (1971) (1971 and 1979 are the only reported years). In a detailed sectoral study of the long-term structure of the U.S. manufacturing industries, Mohr (1980) found that concavity conditions were violated by the data for most industries. Results of Moroney and Trapani (1981) do not satisfy the regularity conditions at almost all sample points when Hicks-neutral technical change is not imposed. Interestingly enough Wills (1979) reports that in his study of technical change in the U.S. primary metals industry, concavity conditions were usually not satisfied under non-neutral technical change but were met when neutrality was imposed.

Researchers have adopted a number of approaches for dealing with non-concavity problems. Lau (1978) and Gallant and Golub (1984) have succeeded in developing general computational methods to estimate a flexible functional form constrained to respect concavity conditions. As Diewert and Wales (1984) have pointed out, these methods only force local concavity and moreover do not yield satisfactory results. Hence, they have developed a method which consists of imposing global concavity on some flexible functional forms without destroying the flexibility property. The approach requires a good deal of additional work from a modelling point of view. A much simpler approach was used by Caves, Christensen and Swanson (1981). Their method consists of checking if the parameter generating the positive own-price elasticity is significantly different from a value of the parameter that would generate non-positive elasticity estimates. If the two parameter values are not significantly different then the authors conclude that violation of the regularity conditions has no real consequences.[16] As can be seen in Table 2, using their method of correction five sectors (forestry, construction,

16. See Caves and Christensen (1980) p. 424-25 for the theoretical justification of such procedure.

manufacturing durables and non-durables and utilities) still violate concavity conditions for the two selected years at the 0.01 level of significance. For the remaining sectors, the observed violation of the regularity conditions has no practical significance.

b) Patterns in Substitution Possibilities

In general the results suggest that the demands for the different factors of production are not very sensitive to price variations. Indeed virtually all own-price elasticities are well below one.

With respect to substitution among inputs, the overall impression is that the substitution possibilities are considerable. Of 204 values of cross-elasticities of substitution presented in Table 2, 125 have a positive sign (64 for 1971 and 61 for 1979) and 52 are greater than one (25 for 1971 and 27 for 1979). However the substitution possibilities between the two types of depreciable capital are more limited. Structures and equipment are found to be complements in all sectors except agriculture, transportation and communication (1971 only), utilities and mining.

Another interesting relationship is that between the two types of depreciable capital and labour. Structures and labour are complements in forestry, construction, manufacturing durables (1979), transportation and communication (1979). In the remaining sectors they are usually substitutes and in the manufacturing non-durables sector, the point estimates of the elasticity of substitution are 2.3 and 3.1 in 1971 and 1979 respectively. Similarly, equipment and labour are substitutes in forestry, the two manufacturing sectors, trade, transportation and communication and mining.

Our results also imply that labour and materials are substitutes in all sectors except utilities. Equipment and materials are substitutes in all industries except the two manufacturing categories and mining while structures and materials are complementary in agriculture, manufacturing durables and non-durables, trade and utilities.

Given the novelty of including materials inventories as a factor of production, it is worthwhile examining the potential substitution relationships with other inputs. A decline in the desired stock of inventories is likely to be associated with a decline in structures, since less warehouse space would be required. On the other hand, the demand for equipment might be expected to rise as moving materials from suppliers to users and within the user's plant become more important. Part of this increased demand for equipment would be reflected in a rise in purchases of transportation services to the extent that transportation equipment is not owned by suppliers or users of materials. Given the close but not exact stock-flow relationship between materials inventories and materials (purchased service inputs are not included in inventories), complementarity might be expected between the two inputs. However, to the extent that a decline in the desired stock of inventories raises the demand for transportation services, which are included in materials but not material inventories, the two inputs would be substitutes.

What do the data tell us about the substitution relationships between materials and other inputs? Of the eight sectors for which inventory data are available, structures and inventories are substitutes only in agriculture, manufacturing durables and transportation and communication for both selected years. Equipment and inventories are highly substitutable in agriculture, construction, manufacturing non-durables and FIRE but less so in trade. They are complements in manufacturing durables, transportation and communication (1979) and utilities. Labour and inventories are substitutes in all sectors but manufacturing and transportation and communication. Finally, materials and inventories are complements in five out of the eight industries. They are substitutes in trade, utilities and transportation and communication.

One of the most important debates in empirical production economics concerns the relationship between aggregate capital and energy in manufacturing. Engineering and econometric studies of energy-capital substitution possibilities often give contradictory results. Engineering analyses typically conclude that substitution is the prevalent pattern. Most of the econometric studies based on

cross-section data confirm the engineering results. On the other hand time-series studies (static or dynamic) often indicate complementarity. However, recent time-series results for Canada indicate that capital and energy are substitutes.[17] Our results offer partial support for this conclusion. Energy and equipment show a clear substitution pattern in the durable and non-durable manufacturing sectors and although structures and energy are substitutes in durables manufacturing, they are complements in non-durable manufacturing.

It is sometimes suggested that models of the sort used in this paper capture the short-run relationship between energy and capital because of the assumption that adjustment to relative price changes is completed within a year. Capital and energy could be complements in the short-run if an increase in the price of energy made energy-intensive capital economically obsolete. However, the data used in this study, and in most other studies, would not capture relative-price-induced obsolescence since the depreciation rate of capital is fixed.

Capital and energy could also be complements in the short run if the energy intensity of new capital is quasi-fixed. In this case a price-induced reduction in the demand for energy would result in less investment in the short-run. Some empirical evidence is provided by Denny, Fuss and Waverman (1979) on this issue. The results from their static model (estimated for the 1962-73 time period) indicate that aggregate capital and energy are substitutes in manufacturing. However 8 of the 19 manufacturing industries estimated from 1962 to 1975 using a dynamic version of the model show short-run capital-energy complementarity, most of them concentrated in manufacturing non-durables. On the other hand, results presented in section 6 of this paper indicate that capital-energy complementarity is found only when inventories of raw materials are included as a factor of production. This suggests that inadequate modelling of the adjustment process is not the only reason for finding structures-energy complementarity in non-durable manufacturing.

17. As quoted in OECD (1982).

At this point, it is worth noting that including materials inventories in the analysis makes significant changes in substitution patterns in manufacturing. It reduces substitution possibilities between structures and energy but enhances substitutability between equipment and energy. These changes indicate that even if inventories are a small proportion of total costs, they seem to introduce an important new dimension in producer cost-minimizing behaviour. This issue will be discussed in the section on models comparison.

c) Bias in Technical Change

As can be seen in Table 3, technical progress is found to be non-neutral in all sectors. The results suggest structures-using technical change in all sectors except two: agriculture and transportation and communication, where it is neutral. In agriculture, forestry and utilities, technical change is neutral for equipment whereas it is equipment using for the remaining industries. These results are somewhat different from those of Rao and Preston (1983), who found technical change to be neutral or saving with respect to capital.[18] For three out of ten industries, technical progress is significantly labour using while it is labour saving in agriculture, construction, manufacturing durables and mining. Technical change is materials using in agriculture, forestry and construction and manufacturing non-durables. Finally, technical change is inventory saving in agriculture, construction, and transportation and communication. The inclusion of inventories of materials does not seem to lead to different patterns in technical change except in transportation and communication where the bias is reversed for every factor of production.

The implications of bias in technical change for the impact of factor price changes on technical progress are worth emphasizing. For example, if technical change is equipment using then the increase in the stock of equipment that would accompany a decrease in its price (ceteris

18. Changes in the definitions of the new cost of capital and total costs as well as a longer sample period all contribute to this difference.

Table 3

BIAS IN TECHNICAL CHANGE[1]

	Structures	Equipment	Labour	Inventories	Energy	Materials
Agriculture	N (=0)	U (>0)	S (<0)	U (<<0)	-	U (<<0)
Forestry	U (>>0)	N (=0)	N (=0)	-	-	U (<<0)
Construction	U (>>0)	U (>>0)	S (<<0)	S (<<0)	-	U (<0)
Manufacturing durables	U (>>0)	U (>>0)	S (<<0)	U (>>0)	U (>>0)	S (<0)
Manufacturing non-durables	U (>>0)	U (>>0)	N (=0)	N (=0)	S (<<0)	U (>0)
Trade	U (>>0)	U (>>0)	U (>0)	N (=0)	-	S (<<0)
Transp. and Comm.	N (=0)	U (>>0)	U (>0)	S (<<0)	-	S (<<0)
Finance, Insurance and Real Estate	U (>>0)	U (>>0)	U (>>0)	U (>>0)	-	S (<<0)
Utilities	U (>>0)	N (=0)	U (>>0)	N (=0)	-	S (<<0)
Mining	U (>>0)	U (>>0)	S (<<0)	-	-	N (=0)

(1) N: neutral, S: saving and U: using. (=0) indicates that the relevant parameter is not significantly different from zero at the 0.95 confidence interval for a one-tail test; (>>0) [(>0)] indicates significantly greater than zero at the 0.975 [0.95] confidence interval for a one-tail test. Similarly (<<0) [(<0)] indicates significantly less than zero at the 0.975 [0.95] confidence interval for a one-tail test.

paribus) would increase technical progress.[19] In general, if technical change is non-neutral then relative price movements, arising from policy changes or other sources, influence technical progress.

19. See footnote 9.

5. Separability Tests

a) Testing the existence of a consistent aggregate for capital

The results presented above suggest that disaggregating depreciable capital is worthwhile in the sense that it allows the impact of changes in the cost of equipment or structures to be calculated more accurately. However, if all cross elasticities involving equipment or structures and other inputs were one (or the same), then there would be little benefit from disaggregating capital. The formal joint hypotheses to be tested are $\sigma_{KS_L} = \sigma_{KS_M} = \sigma_{KS_I} = \sigma_{KE_L} = \sigma_{KE_M} = \sigma_{KE_I} = 1$ and $\sigma_{KS_L} = \sigma_{KS_M} = \sigma_{KS_I} = \sigma_{KE_L} = \sigma_{KE_M} = \sigma_{KE_I}$. This of course tests for weak separability[20] (linear and nonlinear) of the capital inputs and hence the existence of a consistent aggregate index for capital. So the null hypothesis considered is $Y = F(K,L,M,I,Q,t)$ where K pools both structures and equipment. If we cannot reject this hypothesis, then it would be legitimate to aggregate the capital stock and estimate the function with this new capital stock.

Following the lines of Berndt and Christensen (1973), the testing framework for separability conditions is structured as follows. We first test the restrictions for global separability.[21] In this case

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20. The concept of separability is due to Leontief (1947) and Sono (1961). Note that energy is considered as a separate input in the manufacturing sectors.

21. To test the global separability hypothesis, we have to compute the change in the weighted sum of squared residuals from imposing the restrictions $\gamma_{ij} = 0$, $i = K^S, K^E, L, M, I$. This change in the weighted sum of squared residuals is divided by the number of restrictions. This ratio is then divided by the ratio of the weighted sum of squared residuals without imposing the restrictions upon the number of residual degrees of freedom. The resulting test (which may not be as powerful as desired because of the relatively small sample) is distributed asymptotically as a F test which is itself distributed asymptotically as chi-squared divided by its number of degrees of freedom with restrictions imposed. These test statistics are asymptotically equivalent to likelihood ratio test statistics which we have not been able to compute directly because the software used (TROLL) does not provide the maximum of the likelihood function. Other separability tests are conducted in a similar manner.

the translog function reduces to a logarithmic Cobb-Douglas function. If we cannot reject the hypothesis, then we accept that all cross elasticities of substitution are unity and this terminates the testing exercise. If we reject global separability, we test for weak separability. If both weak separability hypotheses are rejected we conclude that an aggregate stock of capital (structures and equipment) does not exist. Table 4 presents the statistical thresholds used to test the three separability hypotheses.

The calculated chi-squared for the separability tests are presented in Table 5. The global separability hypothesis is not rejected for agriculture and mining. In the remaining sectors, it is strongly rejected so we pursue the testing procedure for these sectors. The linear separability conditions are rejected for all the remaining sectors except forestry and manufacturing durables. The milder nonlinear separability restrictions are rejected for the six remaining sectors. However Blackorby, Primont and Russell (1977) have noted that the Berndt-Christensen separability procedure is biased in favour of rejecting the hypothesis of weak separability. Consequently the rejection of the non-linear separability conditions would be more doubtful for manufacturing non-durables.

We can therefore draw the conclusion that it is important to disaggregate the stock of capital in construction, manufacturing non-durables, trade, transportation and communication, FIRE and utilities. In the remaining sectors, the use of an aggregate stock of capital is justified but has the disadvantage of ignoring the substitution possibilities between structures and equipment. For example structures and equipment are highly complementary in forestry and strongly substitutable in the mining industry.

b) Testing the existence of a real value-added output specification

A real value-added function can only be used to study the structure of production if labour and capital are weakly separable from intermediate inputs. Berndt and Wood (1975) concluded that, for the

Table 4

Critical Values for Separability Testing of
Y=F(K,L,M,I,Q,t) with respect to structures, equipment
and value - added

	Type of Separability[1]	Residual degrees of freedom	Restrictions	F distribution (α=0.01) critical value	Chi-Squared distribution (α=0.01) critical value
Agriculture	G	70	10	2.59	2.32
Construction	L	73	7	2.75	2.64
Trade	NL	74	6	2.89	2.80
Transp. and Comm.	VA	71	9	2.67	2.41
Fire					
Utilities					
Manuf. durables	G	85	15	2.26	2.04
Manuf. non-durables	L	91	9	2.61	2.41
	NL	92	8	2.71	2.51
	VA	86	14	2.30	2.08
Forestry	G	72	6	3.06	2.80
Mining	L	73	5	3.28	3.02
	NL	74	4	3.58	3.32
	VA	73	5	3.28	3.02

(1) G: Global; L: linear; NL: nonlinear; VA: value - added.

Table 5

Calculated Chi-squared of Separability Restrictions

Sectors	Global Separability	Weak Separability		Value-added Separability
		linear	nonlinear	
Agriculture	1.47	-	-	4.18
Forestry	42.15	2.41	-	28.00
Construction	12.81	4.03	5.11	52.40
Manuf. durables	37.54	2.70	-	36.49
Manuf. non-durables	59.17	18.92	3.07	62.71
Trade	143.03	8.18	17.97	223.38
Transp. and Comm.	77.79	95.77	110.05	250.51
Fire	22.35	10.27	12.99	36.36
Utilities	156.48	137.38	28.62	242.37
Mining	2.72	-	-	1.57

U.S., the conditions for value-added separability were not satisfied. Denny and May (1977) tested the existence of a real value-added function in Canadian manufacturing. Based on a translog production function, they rejected separability of capital and labour from other inputs and concluded that the use of a real value-added function in studies of the structure of production is inappropriate. The capital-labour relationship cannot be estimated independently from other factors. We performed similar tests for our ten-sector disaggregation. The results of this exercise are presented in the last column of Table 5. Except in mining, we strongly reject the hypothesis that capital, labour and inventories are separable from intermediate inputs.

6. Comparison with other models

Although there are few multi-sectoral studies in which equipment and structures are separated, such a disaggregation is more common in studies of the manufacturing sector. A comparison of the estimated cross elasticities involving equipment and structures is provided in Table 6. With respect to the elasticity of substitution between equipment and structures, the point estimates in our study indicate complementarity, which was also found by Woodland (1975) and Berndt and Watkins.[22] In the other studies identified in Table 6, equipment and structures are substitutes, and strongly in the Berndt-Christensen study.[23]

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22. In Berndt and Watkins (1981) it is noted that structures and equipment are complements but details regarding the elasticities of substitution with respect to structures and equipment are not reported. The underlying model used by Berndt and Watkins constrains the long-run elasticity of substitution between equipment and structures to zero.

23. The analysis of a three-input production function by Berndt and Christensen (1973) demonstrated inter alia that structures and equipment were a consistent aggregate for capital in U.S. manufacturing 1929-68. In contrast Boddy and Gort (1971) rejected the hypothesis of equipment-structures separability.

Table 6

Estimated Elasticities of Substitution in the Manufacturing Sector

	Delorme-Lester[1]			Denny-Fuss[3]	Berndt-Christensen[4]	Mohr[5]
	durables	non-durables	Woodland[2]			
$\sigma_K S_K E$	-2.021	-3.019	-0.421	0.768	6.556	0.924
$\sigma_K S_L$	-0.738	3.149	0.205	0.764	1.518	0.967
$\sigma_K E_L$	1.121	1.027	0.346	0.748	1.341	0.988
$\sigma_K S_M$	-0.634	-0.603	-	-	-	1.000
$\sigma_K E_M$	-0.398	-0.617	-	-	-	0.903

(1) 1979, Canada.

(2) 1961, Canada.

(3) average of 1929-68, U.S.

(4) average of 1929-68, U.S.

(5) average of 1957-72, U.S.

Labour and equipment are substitutes in all studies reported in Table 6. Labour and structures are complements in manufacturing durables in our study, but are substitutes in non-durables. In all other studies labour and structures are substitutes. Finally, our econometric exercise leads to the conclusion that materials are complementary with structures and equipment, which contradicts the results of Mohr (1980) for the U.S.

It is of considerable interest to compare our results with a similar Canadian study conducted by Rao and Preston (1983). The Rao-Preston model was reestimated with some modifications. Their study uses the basic KLEM framework allowing for non-homotheticity and non-neutral technical change and covers the 1957-79 period. As a first step we updated their analysis enlarging the sample to 1982 and used new data on the user cost of capital. In their original model Rao and Preston

Table 7

Impact of Capital Stock Disaggregation and Inventories of Raw Materials on Selected Elasticities of Substitution, 1979[1]

	σ_{KL}	σ_{K^SL}	σ_{K^EL}	σ_{KM}	σ_{K^SM}	σ_{K^EM}	σ_{KE}	σ_{K^SE}	σ_{K^FE}	σ_{LE}	σ_{LM}	$\sigma_{K^SK^F}$
Agriculture												
Rao-Preston	-0.00	-	-	1.19	-	-	-	-	-	-	0.85	-
Model #1	-0.39	-	-	0.35	-	-	-	-	-	-	0.89	-
Model #2	-	-0.14	0.03	-	-0.64	0.63	-	-	-	-	0.67	1.56
Delorme-Lester	-	0.77	-0.48	-	-0.43	0.31	-	-	-	-	0.59	0.36
Forestry												
Rao-Preston	8.33	-	-	-0.90	-	-	-	-	-	-	0.23	-
Model #1	1.69	-	-	-1.34	-	-	-	-	-	-	0.47	-
Model #2	-	-1.62	1.56	-	0.85	0.61	-	-	-	-	0.15	-19.80
Delorme-Lester	-	-0.15	1.90	-	0.03	0.59	-	-	-	-	0.73	-20.00
Construction												
Rao-Preston	-1.83	-	-	1.21	-	-	-	-	-	-	0.67	-
Model #1	-0.20	-	-	0.90	-	-	-	-	-	-	0.29	-
Model #2	-	-2.05	-0.61	-	2.16	1.22	-	-	-	-	0.41	-6.20
Delorme-Lester	-	-2.12	-0.48	-	2.89	0.77	-	-	-	-	0.56	-23.07
Manufacturing Durables												
Rao-Preston	1.00	-	-	1.81	-	-	0.30	-	-	-0.01	0.18	-
Model #1	0.76	-	-	-0.43	-	-	2.09	-	-	0.71	0.32	-
Model #2	-	-0.20	1.05	-	-0.57	-0.38	-	5.97	0.99	0.62	0.32	-1.31
Delorme-Lester	-	-0.74	1.12	-	-0.63	-0.40	-	4.72	1.53	0.65	0.38	-2.02
Manufacturing Non-Durables												
Rao-Preston	1.30	-	-	1.06	-	-	2.30	-	-	-0.03	0.09	-
Model #1	1.79	-	-	-0.53	-	-	2.51	-	-	0.22	0.23	-
Model #2	-	3.20	1.24	-	-0.66	-0.46	-	1.01	3.14	0.23	0.23	-3.58
Delorme-Lester	-	3.15	1.03	-	-0.60	-0.62	-	-2.00	3.40	0.62	0.23	-3.02
Trade												
Rao-Preston	1.82	-	-	1.62	-	-	-	-	-	-	-0.03	-
Model #1	0.09	-	-	-0.08	-	-	-	-	-	-	0.15	-
Model #2	-	-0.11	0.11	-	-0.19	0.04	-	-	-	-	0.14	0.05
Delorme-Lester	-	0.07	0.22	-	-0.21	0.15	-	-	-	-	0.16	-2.80
Transportation and Communication												
Rao-Preston	2.00	-	-	1.38	-	-	-	-	-	-	0.27	-
Model #1	-0.26	-	-	0.01	-	-	-	-	-	-	0.30	-
Model #2	-	-0.93	0.38	-	0.22	0.29	-	-	-	-	0.50	1.32
Delorme-Lester	-	-0.35	0.30	-	0.14	0.31	-	-	-	-	0.50	-0.15
FIRE												
Rao-Preston	1.48	-	-	0.42	-	-	-	-	-	-	-0.68	-
Model #1	0.55	-	-	0.02	-	-	-	-	-	-	-0.03	-
Model #2	-	0.25	-1.45	-	0.44	0.97	-	-	-	-	0.21	-1.15
Delorme-Lester	-	0.10	-0.70	-	0.61	0.32	-	-	-	-	0.10	-1.26
Utilities												
Rao-Preston	1.85	-	-	0.61	-	-	-	-	-	-	-0.90	-
Model #1	0.26	-	-	-0.20	-	-	-	-	-	-	0.48	-
Model #2	-	0.69	-1.81	-	-0.39	1.17	-	-	-	-	0.36	0.38
Delorme-Lester	-	0.27	-0.45	-	-0.35	0.94	-	-	-	-	-0.06	1.29
Mining												
Rao-Preston	2.25	-	-	0.40	-	-	-	-	-	-	0.60	-
Model #1	0.89	-	-	0.50	-	-	-	-	-	-	1.34	-
Model #2	-	1.13	0.96	-	0.27	-0.64	-	-	-	-	0.83	1.85
Delorme-Lester	-	0.54	1.61	-	0.26	-0.79	-	-	-	-	1.10	0.92

(1) The Rao-Preston model covers the 1957-79 period and are calculated using share values provided by S. Rao. Model #1 and Model #2 are estimated from 1957 to 1982 while model #3 is estimated for the 1961-80 sample period.

(1983) used the value of gross output as a proxy for total cost.[24] For the reasons stated in section 2 we amended their model to permit a variable markup by defining total costs as the sum of payments to factors of production using an ex ante measure of the cost of capital.

After the estimation of the modified Rao-Preston model (labeled model #1), we estimated another variant of the model where the stock of capital is disaggregated into structures and equipment (model #2) still for the 1957-82 sub-period. Results for the basic model presented in this paper, which includes materials inventories, are reproduced in Table 7 and are labelled Delorme-Lester. This procedure permits partial isolation the impact of 1) the disaggregation of the capital stock and 2) the introduction of inventories of materials in the analysis. However, it is to be noted that the sample period varies across models.

The key conclusions to be drawn from an inspection of Table 7 are:

(i) In all sectors the reestimation of the Rao-Preston model leads to significant changes in the values of substitution elasticities.

(ii) In many sectors disaggregating the capital stock into structures and equipment alters significantly the implicit elasticity of substitution between aggregate depreciable capital and other inputs.

(iii) Including inventories of materials modifies substantially the values of the elasticities of substitution among some inputs in all sectors.

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24. In order to respect the adding up constraint the capital stock, and hence the total payment to capital, is residually determined.

This last point suggests that the inclusion of inventories of materials in the analysis is important. This assumption can be statistically tested by constraining all coefficients on inventories to zero. The calculated F test far exceeds the critical value for the 0.001 level of significance and we then strongly reject this assumption for all sectors. This latter result is intuitively understandable since, despite their small shares out of total cost, inventories of materials have large variances.

7. Concluding Remarks

The results obtained from econometric analysis of the sort undertaken in this paper should be interpreted with some care for the following reasons. First, aggregation biases are likely to be present. For example, factor shares can shift over time simply because of changes in the mix of goods produced in an industry. Berndt (1976) and Miller (1986) have demonstrated that failure to take into account changes in the product mix biases time series studies towards complementarity for the capital-energy relationship. Disaggregation is a partial solution to this problem but even at the most detailed level for which data are available the product mix is not likely to be constant. Furthermore changes in the nature of relatively narrowly defined products would also bias the results. An example is the "down-sizing" in the automobile industry, which may have, in itself, changed factor shares. The aggregation of industries having quite different technologies could also introduce a bias in the results. For example, an industrial classification combining industries with substantial differences in energy use is likely to bias the empirical results.

Second, the implicit hypothesis in the estimating model is that firms are in long-run equilibrium; this is unlikely to be the case. Prices do not adjust sufficiently to clear markets in the short-run and there are lags in adjusting to changes in relative prices when they do occur. Adjustment of the capital stock is likely to be particularly slow, but it may not be reasonable to consider labour a completely variable factor of production. Consequently, both labour and capital

could be considered quasi-fixed. Adjustment of these quasi-fixed factors is not costless, and these costs should be taken into account. However, identifying the problem is much easier than resolving it. Dynamic models incorporating adjustment costs currently available also have shortcomings and may not result in an improvement in terms of reliability of estimates.[25]

Third, quite different results could be obtained using an alternative functional form or if profit rather than cost functions were estimated. Determining the best approach given the range of models and functional forms available is not an easy task.[26]

Despite these caveats the results presented in this paper provide useful information on the structure of production in Canadian industries. The key findings of the paper are as follows:

1. Homotheticity and neutral technical change are rejected in all industries. Technical change is capital using in almost all industries.

2. The demand for factor inputs is generally price inelastic.

3. Structures and equipment are complements in most sectors while labour and the two types of depreciable capital are substitutes in most industries. Energy and depreciable capital are generally substitutes in manufacturing.

4. Capital is separable from other inputs in only four of ten industries examined. The existence of a value-added function is rejected in all sectors except mining.

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25. See Delorme (1986) for a critical review of dynamic models.

26. See Barnett and Lee (1985) on the functional form issue.

5. The inclusion of materials inventories as a factor of production is important since it adds to the explanatory power of the equations and results in substantial changes to substitution patterns among other inputs.

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